M2PM5 METRIC SPACES AND TOPOLOGY
SPRING 2017

PROBLEM SHEET 3

Exercise 1. Suppose that X is an infinite set equipped with the cofinite topology.

(1) Let $A \subset X$ be a finite set. Compute $\overline{A}$.

(2) Let $A \subset X$ be an infinite set. Compute $\overline{A}$.

Exercise 2. Let X be a non-empty set and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be topologies on X. Must $\mathcal{T}_1 \cap \mathcal{T}_2$ be a topology on X? Must $\mathcal{T}_1 \cup \mathcal{T}_2$ be a topology on X?

Exercise 3. Let A be a subset of the topological space X. Show that $\partial A = \overline{A} \cap \overline{X \setminus A}$.

Exercise 4. Let X and Y be topological spaces. Let:

$$\mathcal{B} = \{U \times V : U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$$

The *product topology* on $X \times Y$ is

$$\mathcal{T} = \{Z \subset X \times Y : Z \text{ is a union of elements of } \mathcal{B}\}$$

This is a topology on $X \times Y$. (You do not need to show this.)

(1) Show that the projection maps:

$$p_1 : X \times Y \to X \qquad\qquad p_2 : X \times Y \to Y$$
$$(x, y) \mapsto x \qquad\qquad (x, y) \mapsto y$$

are continuous, where $X \times Y$ is given the product topology.

(2) Let Z be a topological space. Show that a map $f : Z \to X \times Y$ is continuous if and only if $p_1 \circ f$ and $p_2 \circ f$ are continuous.