M2PM5 METRIC SPACES AND TOPOLOGY
SPRING 2017

PROBLEM SHEET 4

Exercise 1.

(1) Let X be a non-empty set equipped with the discrete topology. Show that X is compact if and only if X is finite.

(2) Let X be a topological space, and let Y and Z be compact subsets of X. Show that $Y \cup Z$ is compact.

Exercise 2. Let X be a compact topological space and let $f : X \to Y$ be a continuous map of topological spaces. Show that $f(X)$ is compact.
(*This would make an excellent exam question. If you can do it, then you are probably getting the hang of compactness.*)

Exercise 3. Let X be a compact topological space. Suppose that, for each $n \in \mathbb{N}$, V_n is a closed non-empty subset of X and that:

$$V_0 \supseteq V_1 \supseteq V_2 \supseteq \cdots$$

Show that:

$$\bigcap_{n \geq 0} V_n \neq \varnothing$$

Is this statement true without the compactness hypothesis?

Exercise 4. Suppose that X is a topological space and that A, B are connected subsets of X such that $A \cap \overline{B} \neq \varnothing$. Show that $A \cup B$ is connected.