Exercise 12.11 Give either a proof of, or a counterexample to, each of the following.
(a) Suppose that X, Y are spaces with subsets A, B. Suppose that neither $X \setminus A$ nor $Y \setminus B$ is connected. Then $X \times Y \setminus (A \times B)$ is not connected.
(b) Suppose that A, B are subsets of a space X and that both $A \cap B$ and $A \cup B$ are connected. Then A and B are connected.
(c) Suppose that A, B are closed subsets of a space X and that both $A \cap B$ and $A \cup B$ are connected. Then A and B are connected.

Exercise 13.4 Which of the following subsets of $\mathbb{R}$, $\mathbb{R}^2$ are compact?
(i) $[0, 1)$;
(ii) $[0, \infty)$;
(iii) $\mathbb{Q} \cap [0, 1]$;
(iv) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$;
(v) $\{(x, y) \in \mathbb{R}^2 : |x| + |y| \leqslant 1\}$;

(vi) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$;
(vii) $\{(x, y) \in \mathbb{R}^2 : x \geqslant 1, 0 \leqslant y \leqslant 1/x\}$.

Exercise 13.14 Suppose that X is a compact metric space with metric d and that $f : X \to X$ is a continuous map such that for every $x \in X$, $f(x) \neq x$. Prove that there exists $\varepsilon > 0$ such that $d(f(x), x) \geqslant \varepsilon$ for all $x \in X$.
[Hint: Show that the map $g : X \to \mathbb{R}$ defined by $g(x) = d(f(x), x)$ is continuous so attains its bounds.]

Exercise 14.11 Suppose that X is a sequentially compact metric space and we are given a nested sequence $V_1 \supseteq V_2 \supseteq \ldots$ of non-empty closed subsets of X. Prove that $\bigcap_{n=1}^{\infty} V_n \neq \emptyset$.

Exercise 16.8 Suppose that (f_n) converges uniformly on D to a function f and that f_n is uniformly continuous on D for each n (recall Definition 13.23). Prove that f is uniformly continuous on D.

Exercise 17.2 Which of the following are complete?
(i) $\{1/n : n \in \mathbb{N}\} \cup \{0\}$, (ii) $\mathbb{Q} \cap [0, 1]$, (iii) $\{(x, y) \in \mathbb{R}^2 : x > 0, y \geqslant 1/x\}$.

Exercise 16.9★ (Dini's Theorem) Suppose that $f_n : X \to \mathbb{R}$ is a continuous function on a compact topological space X and that $f_n(x) \geqslant f_{n+1}(x)$ for all $n \in \mathbb{N}$ and all $x \in X$. Suppose also that (f_n) converges pointwise to a continuous function f on X. Prove that (f_n) converges uniformly on X.